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Fatigue Lifetime Analysis of POM Gears for Generalized Tooth Root Shapes

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Fatigue Lifetime Analysis of POM Gears for Generalized Tooth Root Shapes

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Abstract: The current calculation methods for determining the tooth root load capacity of polymer gears (e.g., VDI 2736) are based on the same assumptions as those for steel gears. However, due to the non-linear material behavior, temperature, and rate dependency of polymers, these predictions are often inaccurate. A previous study employed rate-dependent nonlinear viscoplastic finite element (FE) modelling of polyoxymethylene (POM) to quantify material influences not considered in standard metal gear assumptions. A lifetime model was developed and validated to predict tooth root fracture based on rotational speed for a constant tooth root geometry. In this study, the existing damage model is adapted and validated to include the dependency on notch (tooth root) geometry. The extension of the model to two damage parameters allows for a geometry-independent representation of the nonlinear speed dependency of tooth root breakage. This correlative modelling approach incorporates two independent damage mechanisms inside the material which lead to tooth root breakage failure of the gear. To map these mechanisms, local material states at the crack initiation point are used as damage parameters. Calibration of the bi-parametric damage model with experimental data shows that model predictions fall within the experimental scatter. Further research is ongoing to extend the damage model regarding generalized torque loading conditions.

Keywords: Lifetime Modelling, Polymer Gears, Tooth Root Fracture, Viscoplastic Material Modelling, Finite Element Analysis, Local Damage Criterion

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Introduction

Polymer gears are increasingly popular in various industries due to their low weight, corrosion resistance, quieter operation, lack of lubrication requirements, and cost-effectiveness compared to steel gears. However, current standard calculation methods like VDI 2736, which are based on assumptions for steel gears, are inadequate for polymer gears and lead to inaccurately predicted polymer gear lifetimes [1]. These methods assume rigid body behavior, linear elastic material properties, and rotational speed independent lifetimes, which do not align with the nonlinear, rate-dependent properties of polymers [1-3]. This paper investigates tooth root breakage, a common failure mode in both dry and lubricated applications, occurring when tensile load at the tooth root exceeds the material's fatigue limit [3,4].

Researchers have tried to predict this failure analytically by adjusting current standards [2] or through lifetime modelling consisting of a numerical finite element (FE) simulation combined with fatigue models [5-8]. Common simulation methods use quasi-static FE modeling and assume linear elastic behavior, but these are insufficient for polymers due to their nonlinear and transient material properties and the dynamic nature of gear meshing [9-11]. Previous studies often used steel fatigue models, focusing on strain-based criteria, but these are unsuitable for polymers because of significant differences in material characteristics [12]. In [13], a lifetime modelling approach consisting of a damage model based on an FE model, including a physical material modelling approach utilizing the Three-Network-Viscoplastic (TNV) model [14,15] that is based on the original work by [18,19], showed the ability to predict the lifetime dependency on rotational speed for a constant load torque level and root geometry. The damage model extrapolates the increase of the maximal principal strain at the crack initiation point in the tooth root, based on the values of five simulated loading cycles (revolutions), to the experimental determined lifetime for defining a strain criterion at which the gear fails. The critical strain in the tooth root is increasing continuously as it can be shown for the uniaxial case [16]. This damage modeling approach is based on the physical material behavior and has the capacity to incorporate dynamic effects. However, damage modeling based on extrapolation has the disadvantage that it requires the simulation of 5 consecutive cycles and that the extrapolation is sensitive to these 5 simulated support points when extrapolating to cycle numbers over 10^5. Thus, new approaches considering the unique properties of polymers are necessary for accurate prediction of tooth root breakage and fatigue life in polymer gears.

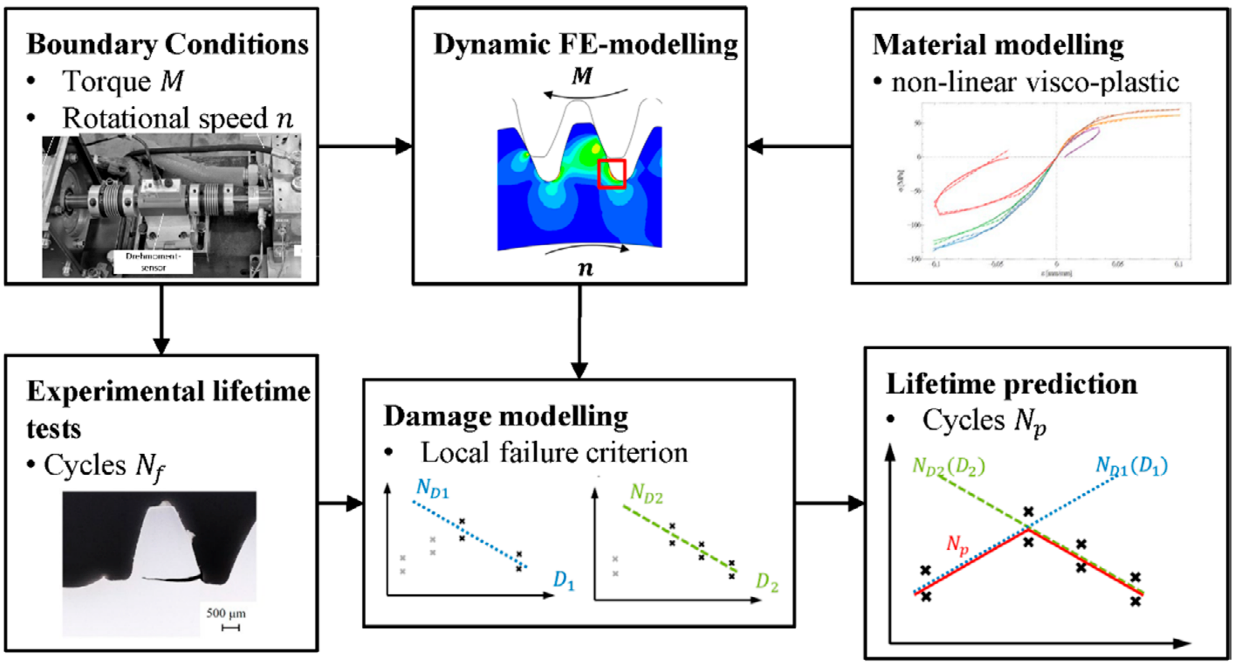

Figure 1. Structure of the lifetime model.

The objective of this study is to generalize the lifetime model from the previous study [13] with the ability to work independently from rotational speed and root geometry at a constant torque level. With regard to the structure of the lifetime model in Figure 1, this implies an adaptation of the damage model, which combines the results from experimental lifetime tests and FE-simulations. Consequently, the experimental lifetime tests conducted in the previous study with the reference tooth root geometry are supplemented with tests of an optimized geometry at the same conditions on the test rig. The tests are conducted under constant loading torque at different rotational speeds with a tempered oil lubrication at 50°C. The dynamic FE-modeling scheme remains unchanged and is still based on physical material modeling with the TNV model. Another objective is to alter the methodology employed in damage modeling from an extrapolative to a correlative approach, as is the case with classical fatigue models [12,17]. This addresses the inherent limitations of the extrapolating approach, whereby minor alterations to the simulated support points result in significant changes in the calculated damage values for high numbers of cycles. Consequently, this approach is highly susceptible to the damage values derived from the initial five revolutions. A correlative approach circumvents this issue and is computationally less costly. This holds, because the damage parameters and the material behavior are assumed to be unchanged in each revolution, thus only one cycle must be simulated.

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Experimental Results

Figure 2 shows the results of the experimental lifetime tests for both tooth root geometries, shown at the right side of the figure. The results of the reference geometry are taken from [13]. The optimized geometry is an adaptation of the reference root shape, which reduces the stress concentrations from the notch effect and thus increases lifetime. Figure 2 illustrates that the increase in lifetime is dependent on rotational speed. This can be attributed to time-dependent local material states at different rotational speeds. Compared to the reference geometry, the optimized geometry can be tested up to 3000 min^{-1} without causing a change in the failure mode of the gear (tooth root fracture to temperature induced failure). The lifetime of both geometries is characterized by speed dependency, which results in a local lifetime maximum. As the speed increases, the trend reverses from this maximum, with a subsequent decrease in lifetime. This suggests that there are two opposing effects, i.e. mechanical or thermal destruction of molecular bonds. In other words, two damage mechanisms that are contrary dependent on the dynamics of the load, with one or the other dominating at different speeds. This hypothesis requires the introduction of a second damage parameter.

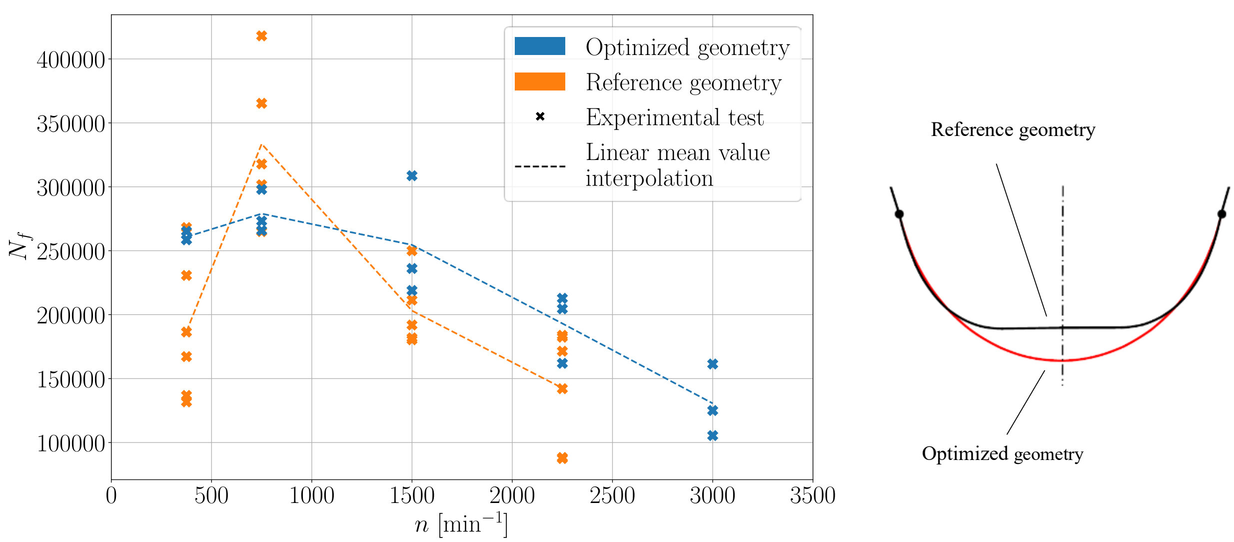

Figure 2. Experimental lifetime tests at constant torque level for two root geometries.

Bi-Parametric Damage Model

The bi-parametric damage model introduced here is based on two independent damage mechanisms that lead to the same failure mechanism (tooth root breakage) of the gear with the corresponding lifetimes N_{D1} and N_{D2}. As they are independent of each other, the first one reached (lower

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one in value) is decisive for the failure and the resulting predicted lifetime N_p of the model. Hence, no cumulative interactions of the damage mechanisms are assumed, see equation 1.

$$N_p = \min(N_{D1}, N_{D2}) \quad 1)$$

Regarding the experimental results, it can be concluded that with increasing rotational speed, the first damage mechanism becomes dominant for lifetime limitation. Once a critical speed n_{crit} is reached, the dominant damage mechanism is altered as illustrated in Figure 3(b). Therefore, for loading conditions at low rotational speed, one mechanism is limiting lifetime, whereas at high rotational speed, the other mechanism is restricting lifetime. Each damage mechanism depends on a damage parameter D_n, which correlates with the corresponding mechanism lifetime N_{D1} and N_{D2}, see Figure 3(a). This dependency is for both mechanisms characterized by the damage function *f*. In general, the type of this function must be monotonically decreasing with increasing damage parameter. In equation 2, the damage function is presented. It is assumed to be linear.

$$N_{Dn} = f(D_n) = a_n \cdot D_n + b_n \quad 2)$$

In order to achieve a contrary speed dependency of the lifetimes N_{D1} and N_{D2} in Figure 3(b), the respective damage parameters D_1 and D_2 must have a contrary dependency on rotational speed. This means that with increasing speed, the damage parameter D_2 will increase and thus lead to a lower lifetime N_{D2}, while the other damage parameter D_1 will decrease and thus the lifetime N_{D1} will increase. This represents the condition for a suitable choice of damage parameters, which must be based on local material states at the crack initiation point in order to ensure a physically motivated correlation.

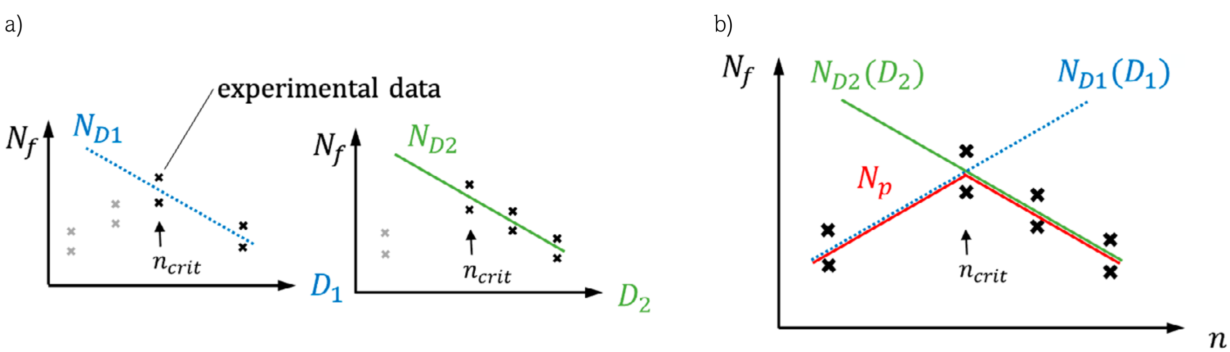

Figure 3. (a) Calibration of damage functions, (b) Evaluation of predicted lifetime N_p.

Damage Parameters

The damage parameter D_1, defined in equation 3, consists of the amplitude of the maximal principal strain scaled with the loading period T to increase the dependency on rotational speed. The strain amplitude describes the difference between the minimal and maximal value of the maximal principal strain in the crack initiation point, which is an indicator of the tension load in the direction of the crack opening. The period T is proportional to time, in which the tooth is loaded. This represents an extension of the damage parameter used in the previous study [13], where the maximal principal strain was used exclusively. Physically, the damage mechanism of D_1 can be interpreted as a mechanical destruction of the polymer's molecular bonds. In contrast, the damage parameter D_2, defined in equation 4, equals the amplitude of the viscoplastic strain energy density rate at the point of crack initiation, indicating how fast energy is dissipated. This is a direct indicator of local heating, hence the mechanism of D_2 can be regarded as a measure for thermal destruction of molecular bonds. Figure 4 shows both damage parameters with their contrary dependency on rotational speed, as the influence of the geometries is a direct result of physical material modeling.

$$D_1 = T \cdot \Delta\varepsilon_1 \tag{3)}$$

$$D_2 = \Delta\dot{U}_{vp} \tag{4)}$$

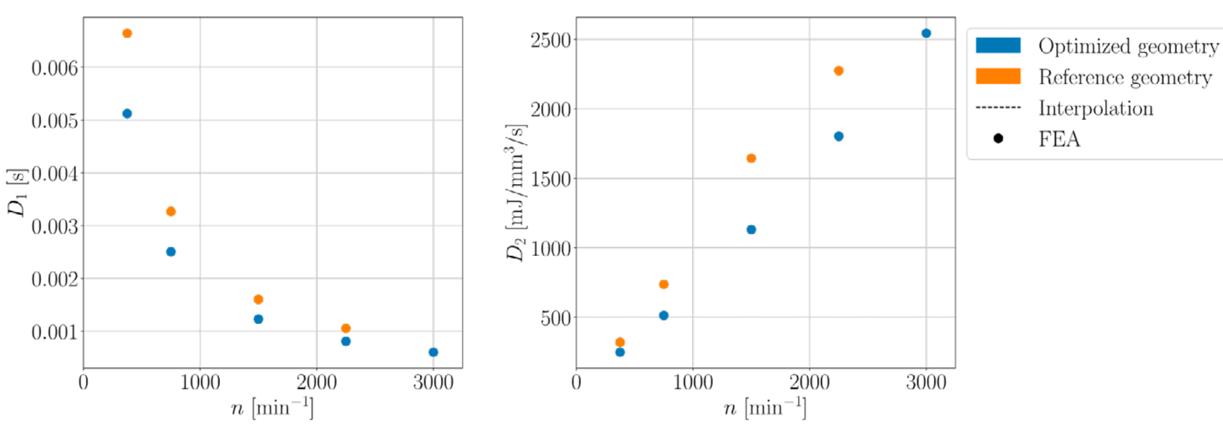

Figure 4. Damage parameters in dependence of rotational speed.

Application to Experimental Results

In order to apply the bi-parametric damage model with the specified damage parameters to the experimental results, the experimental data must first be associated with respect to the governing damage mechanism as shown in Figure 3(b). This step is necessary to calibrate the two damage functions and leads to the condition that both mechanisms are represented in the data by the chosen

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speed sampling. The transition of the mechanisms is defined by the splitting method, which may occur either on or between data points. Here the former one is assumed, hence a critical speed n_{crit} exists, where the damage of both mechanisms is minimized. Figure 5 shows the damage mechanisms lifetimes N_{D1} and N_{D2} predicted by the calibrated damage functions as well as the effective predicted lifetime N_p along the rotational speed. The damage functions of both mechanisms were fitted to the damage parameter data of both geometries, so the criterion is independent of the geometry. The figure shows that the variation of the tooth root geometry influences the mechanisms' lifetimes. This is a result of the change in the damage parameters due to the changed notch shape. Furthermore, two different speed dependencies are evident, N_{D1} multi-linear and N_{D2} linear, which can be attributed to the different speed dependencies of the damage parameters D_1 and D_2. The lifetime prediction N_p of the model reproduces the experimental data within its scatter as well as the local lifetime maxima.

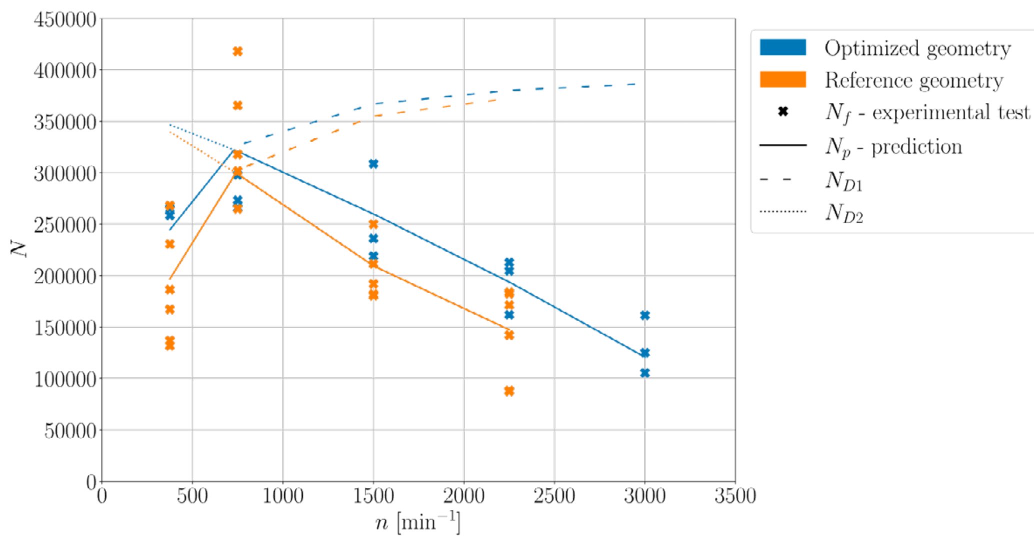

Figure 5. Evaluation of predicted lifetime N_p out of damage mechanisms lifetime.

Conclusion

The tooth root fracture failure mechanism exhibits a speed dependency with a trend reversal, which leads to a local lifetime maximum. The bi-parametric damage model postulates two independent damage mechanisms of the polymer, which can be attributed to this behavior and depend on local material states at the crack initiation point. The first damage parameter is based on the maximum principal strain, which is a direction-dependent variable. It can be interpreted as a mechanical destruction of the polymer's molecular bonds. The second damage parameter considers the thermal destruction of these bonds by considering the dissipation rate. This is a direction-independent variable and can be related to short-term local heating. Although the model does not include any

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interaction between the damage mechanisms, the predictions are within the experimental scatter and the local lifetime maximum can be modeled. Further research is required to investigate the interaction between damage mechanisms and changes in linear or even nonlinear damage functions in order to generalize the damage model for varying loading boundary conditions.

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